MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

BNA3014 –NUMERICAL ANALYSIS

(All sections / Groups)

14 OCTOBER 2017
9 a.m – 11 a.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of **4 pages** including cover page and formulae.
2. Attempt **ALL** questions and write your answers in the Answer Booklet provided.

Question 1 (12.5 marks)

a) Consider the following function,

$$f(x) = \sqrt{x^5 + 10} - 3$$

(i) Suggest a function g(x) that will avoid loss of significance error of f(x).

[2 marks]

(ii) Compute f(0.4) and g(0.4) by using four-digit arithmetic and rounding.

[3 marks]

(iii) Given the actual value is 0.1638963, find the relative error for both calculations in part (ii). **[2 marks]**

b) Find the root of the following equation by using **Newton-Raphson** method with tolerance 1×10^{-4}.

$$f(x) = e^{-x} - x, \qquad \text{starting with } p_0 = 0$$

Note: Write your answer correct to 6 decimal places.

[5.5 marks]

Question 2 (12.5 marks)

a) Given the system $Ax = b$ where

$$A = \begin{bmatrix} 2 & 1 & 5 \\ 4 & 4 & -4 \\ 1 & 3 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 0 \\ 6 \end{bmatrix}$$

(i) When $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$, $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$ and $A=LU$, find the lower-triangular matrix, L and upper-triangular matrix, U. **[5.5 marks]**

(ii) Using **LU Decomposition**, solve the system $Ax = b$. **[4 marks]**

b) Use **Lagrange interpolating polynomial** of second order, $P_2(x)$ to find a polynomial on the basis of the data given below:

$x_0 = 1$; $f(x_0) = 0$
$x_1 = 4$, $f(x_1) = 1.386294$
$x_2 = 6$, $f(x_2) = 1.791760$

Hence, evaluate $P_2(2)$. **[3 marks]**

Continued...

Question 3 (12.5 marks)

a) Solve the following set of equations by using **Gauss Seidel** iteration with tolerance 1×10^{-3} (start with $x = 0, y = 0, z = 0$).

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

Note: Write your answer correct to 6 decimal places.

[6 marks]

b) Fit a third order **Newton's interpolating polynomial** for a function $f(x) = \log x$ using the numbers $x = 8, 9, 11$ and 12. (Round the answers to 7 decimal places).

x	$\log x$	First divided difference	Second divided difference	Third divided difference
8	0.9030900			
9	0.9542425			
11	1.0413927			
12	1.0791812			

(i) Complete the above table.
(ii) Find the cubic Newton polynomial, $P_3(x)$
(iii) Estimate $\log 10$.

[6.5 marks]

Question 4 (12.5 marks)

(a) Find the best least square **linear line** to the x and y values in the following table:

x	1	2	3	4	5	6	7
y	0.5	2.5	2.0	4.0	3.5	6.0	5.5

[6 marks]

(b) Given the following differential equation:
$y' = 4e^{0.8x} - 0.5y$ for $0 \le x \le 4$ with initial condition at $x = 0, y = 2$ and $h = 1$.
Compute the value of y_1 and y_2 (6 decimal places) by using each of the following method:
(i) **Euler's Method**
(ii) **Runge-Kutta Second Order Method**

[6.5 marks]

End of Page

FORMULAE

Taylor Polynomial

$$P_n(x) = \sum_{k=0}^{n} \frac{1}{k!} f^{(k)}(a)(x-a)^k$$

Lagrange Polynomial

$$P_n(x) = \sum_{i=0}^{n} f(x_i) L_i(x)$$

Newton's Polynomial

$$P_n(x) = a_0 + a_1(x - x_0) + \ldots + a_n(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

Newton-Raphson Method

$$p_{n+1} = p_n - \frac{f(P_n)}{f'(P_n)}$$

Euler Method

$$y_{n+1} = y_n + hf(x_n, y_n); \qquad n = 0,1,2,3,\cdots .$$

Runge-Kutta second order methods

$$k_1 = f(x_n, \quad y_n)$$

$$k_2 = f(x_n + h, \quad y_n + hk_1)$$

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2)$$

Simple Linear Regression

$$y = ax + b$$

$$\Sigma y = na + b\Sigma x$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$